Chapter 3

Section 3.4 – Velocity and Other Rates of Change

Recall for Rectilinear Motion (motion along a line):

Position function = $x(t)$

Velocity function = $v(t)$ $= x'(t)$ INSTANTANEOUS RATE OF CHANGE OF POSITION

$v(t) > 0 \Rightarrow$ object moving in positive direction

$v(t) < 0 \Rightarrow$ object moving in negative direction

$v(t) = 0 \Rightarrow$ object stopped/changing direction

$|v(t)|$ = Speed

Average Velocity: from time t_1 to t_2:

$$v_{ave} = \frac{\text{Displacement}}{\text{Time}} = \frac{\Delta x}{\Delta t} = \boxed{\frac{x(t_2) - x(t_1)}{t_2 - t_1}}$$

→

Recall for Rectilinear Motion (motion along a line):

Acceleration function = a(t) $= v'(t) = x''(t)$ → RATE OF CHANGE OF VELOCITY

If a(t) and v(t) have the same sign $\Rightarrow$ speeding up

If a(t) and v(t) have opp. signs $\Rightarrow$ slowing down

DISTANCE / TIME^2

→

Example: $x(t) = 2t^3 - 21t^2 + 60t + 3$ $\quad t \geq 0$

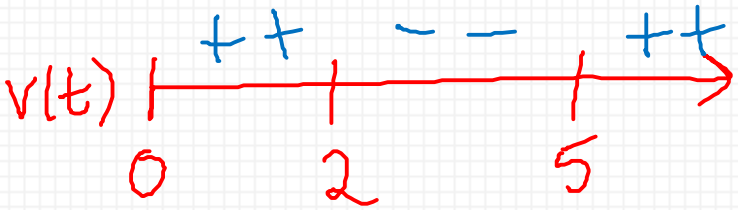
a. When is the object moving left or right? Assume time is in seconds and position is in feet.

$v(t) = 6t^2 - 42t + 60 = 0$

$6(t^2 - 7t + 10) = 0$

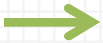
$6(t-5)(t-2) = 0$

$t = 2, 5$



RIGHT: $[0,2) \cup (5,\infty)$

LEFT: $(2,5)$



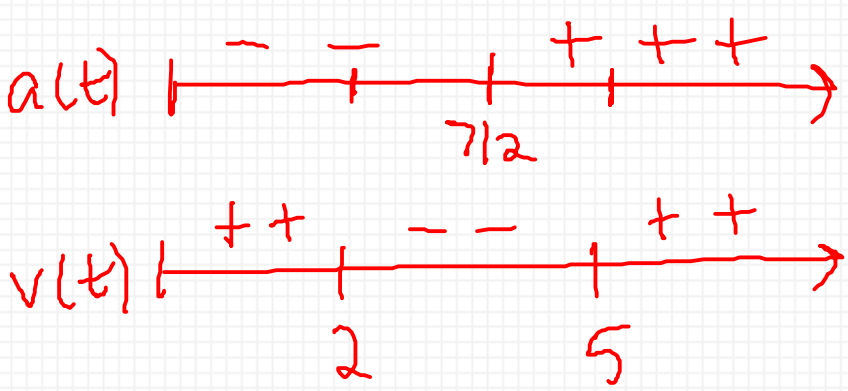
Example: $x(t) = 2t^3 - 21t^2 + 60t + 3$

b. When is the object speeding up or slowing down?

$a(t) = 12t - 42 = 0$

$6(2t - 7) = 0$

$t = 7/2$



SPEEDING UP

$(2, 7/2) \cup (5, \infty)$

SLOWING DOWN

$(0, 2) \cup (7/2, 5)$

Example: $x(t) = 2t^3 - 21t^2 + 60t + 3$

c. Find the displacement over the first 6 seconds.

$\Delta x = x(6) - x(0) = 39 - 3 = \boxed{36 \text{ FT}}$

d. Find the average velocity over the first 6 seconds.

$v_{AVE} = \frac{\Delta x}{\Delta t} = \frac{36 \text{ FT}}{6 \text{ s}} = \boxed{6 \text{ FT/s}}$

Example: $x(t) = 2t^3 - 21t^2 + 60t + 3$

e. Find the total distance traveled over the first 6 seconds.

$x(0) = 3$

$x(2) = 55$

$x(5) = 28$

$x(6) = 39$

$|\Delta x| = 52$

$+$

$|\Delta x| = 27$

$+$

$|\Delta x| = 11$

90 FT

Ex: A ball is thrown vertically from the top of a 40 ft. building with velocity 66 ft/s.

Find the height, velocity, and acceleration functions.

$h(t) = \frac{1}{2}gt^2 + v_0t + h_0$ (INITIAL HEIGHT)

$v(t) = gt + v_0$ (INITIAL VELOCITY)

$a(t) = g$

$\begin{cases} g = -9.8 \frac{m}{s^2} \\ g = -32 \frac{ft}{s^2} \end{cases}$

$h(t) = -16t^2 + 66t + 40$

$v(t) = -32t + 66$

$a(t) = -32$

Ex: A ball is thrown vertically from the top of a 40 ft. building with velocity 66 ft/s.

a. How high will the ball travel?

$v(t) = 0$

$-32t + 66 = 0$

$t = \frac{66}{32} = \frac{33}{16} \approx 2.0625$

$h(2.0625) = 108.0625$ FT

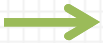
b. What is the impact velocity of the ball?

$h(t) = 0$

$-16t^2 + 66t + 40 = 0$

$t = 4.661$

$v(4.661) = -83.162$ FT/S



Ex: A ball is thrown vertically from the top of a 40 ft. building with velocity 66 ft/s.

c. When will the ball return to its initial position?

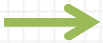
$$h(t) = 40$$

$$-16t^2 + 66t + 40 = 40$$

$$-16t^2 + 66t = 0$$

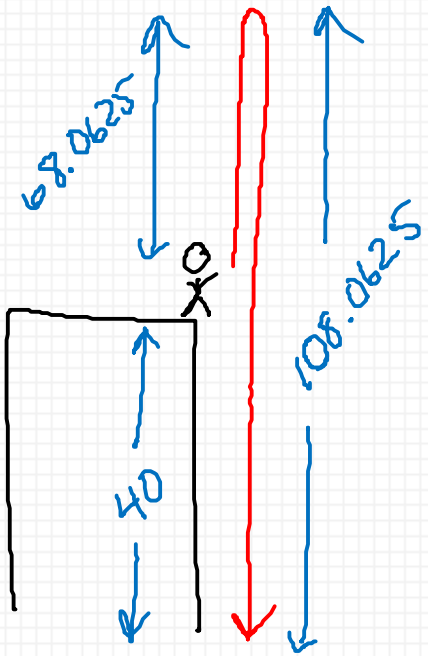
$$t(-16t + 66) = 0$$

$$t = \frac{66}{16} \approx \boxed{4.125 \text{ s}}$$



Ex: A ball is thrown vertically from the top of a 40 ft. building with velocity 66 ft/s.

d. What is the total distance traveled by the ball?



$$\begin{array}{r} 68.0625 \\ +\ 108.0625 \\ \hline \boxed{176.125}\ \text{FT} \end{array}$$

Homework/Classwork:

AP Packet: #85-89, FRQ #1 – 3